A Statistical Approach to the Results of FM Broadcast Frequency Deviation Measurements

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***Abstract*—Supervision of frequency deviation of FM broadcast stations is usually done through measurement of relative time where the frequency deviation limit is exceeded (hereinafter – overshooting time). When assessing these measurement results, it is necessary to take into account that overshooting time is a random variable because its actual value depends on the programme material being translated. Distribution of overshooting time is close to normal and its standard deviation is directly related to the sample mean. This allowed developing the method for evaluating the probability of overshooting the limits on the basis of only one or several overshooting time measurement results.**

Keywords—broadcasting, deviation, limit, measurement, random, distribution, threshold

I. INTRODUCTION

The protection ratios for the planning of FM broadcasting transmitter frequencies are based on the maximum frequency deviation of ±75 kHz. The transmissions exceeding the maximum frequency deviation can cause interference in adjacent channels and to the aeronautical radionavigation service in the frequency band above 108 MHz.

Therefore, the maximum permissible relative overshooting time of frequency deviation (hereinafter – maximum overshooting time) is determined, when frequency deviation may exceed a maximum permitted value f_{max} (often 75 or 77 kHz).The ITU-R SM.1268-3 Recommendation [1] introduces 10^{-4}% as an allowable maximum overshooting time. Administrations of the countries, where frequency resources are sufficient and intervals between FM broadcasting stations are wide enough, use a higher value of maximum overshooting time. For example, regulation of frequency deviation measurement in Lithuania specifies that maximum overshooting time is equal to 1%, whereas in Switzerland it is 10%.

It is not easy to determine compliance of the mentioned deviation parameter with the requirements. Because of a random origin of FM station frequency deviation, the values of overshooting time captured in different measurements will not repeat. In order to be able to conclude (with a certain level of probability) the extent of compliance of frequency deviation with the requirements from just one or several measurement, **we should know the distribution type of overshooting time and its standard deviation (**for this purpose, statistical features of overshooting time measurement results were analyzed).

II. EXPERIMENTAL STUDY

During the experiment, the overshooting time of FM broadcasting stations was measured at fixed monitoring stations in Lithuania and Latvia. The majority of the measurements were carried out by Deva Broadcast FM modulation analyzer DB3000 in Lithuania. Overshooting time of the 8 stations whose sample mean was less than 0.1% was measured by the specialists of the Latvian administration with Audemat-Aztec FM analyzer FM-MC4 because it can measure significant shorter overshooting times than analyzer DB3000. FM stations for measurements were chosen on a random basis; the only requirement was that the overshooting time values should fall into the measurement range of measurement equipment which is being used.

A. Measurement procedure

During the tests, the values of overshooting time were estimated every 15 minutes, as suggested in the aforementioned Recommendation. Thus, a time series of 16 measurement values for every 27 FM station was collected. It took about 4 hours to measure every station.

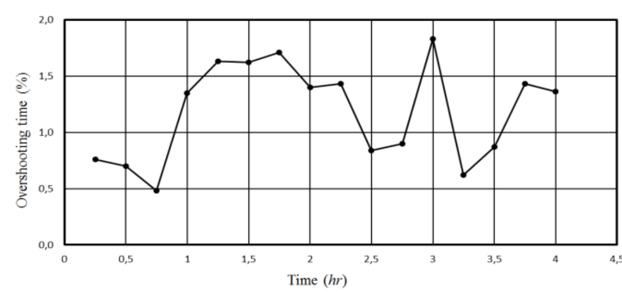

Fig. 1. A typical result of overshooting time measurements.

As an example, the results of overshooting time measurements for one station are presented in Fig. 1. It is evident that, from one measurement, no conclusion on the compliance with the deviation requirements can be made because the results of some measurements exceed the permitted limit of 1% and do not exceed others.

B. Measurement results

A sample mean m and a sample standard deviation s were calculated for each overshooting time series. As a sample mean varies in very wide limits from about 10^{-4}% to 30%, the relation between the sample mean and sample standard deviation of every

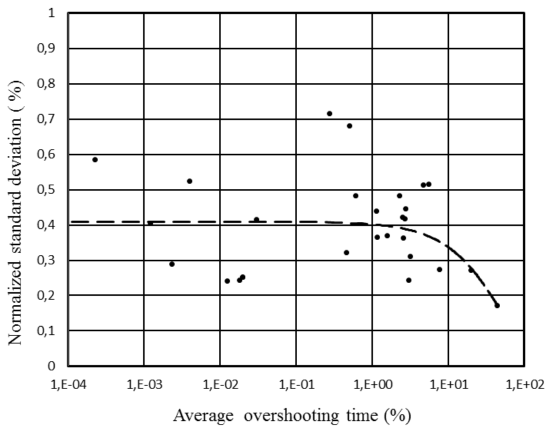

Fig. 2. The dependence of a normalized sample standard deviation on a sample mean of overshooting time. Dots represent measurement results for different stations.

measurement series is better expressed by dependence of a normalized sample standard deviation $s_n=s/m$ on a sample mean m. Such dependence is shown in Fig. 2. We see that if a sample mean is lower than 10%, then a normalized sample standard deviation does not change significantly and its value is around 0.4.

In order to estimate the distribution type of overshooting time, the measurements of seven FM stations in Lithuania were made within several days. A long series of more than 100 overshooting time values for each station was collected during the measurements. Fig. 3 shows that such an amount of values is large enough to determine a series sample mean and a standard deviation with sufficient accuracy.

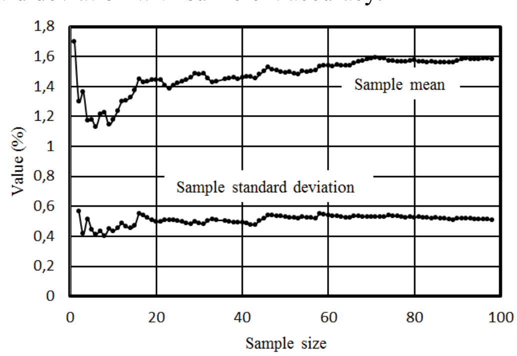

Fig. 3. A sample mean and standard deviation dependence on sample size.

The calculations of a sample moving average and a sample moving standard deviation were performed in order to determine a possible trend. The result of calculations for subsets size=10 is presented in Fig. 4. It shows that there is no linear trend and that only local deviations from mean values for both parameters are observed.

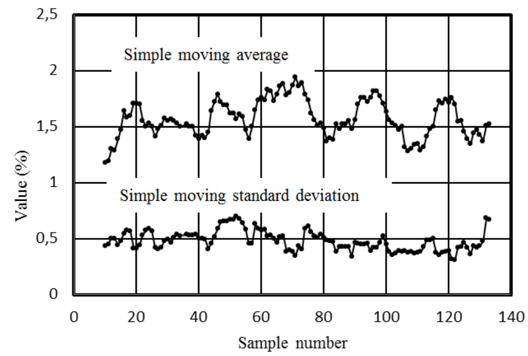

Fig. 4. A graphical plot of the 10-period simple moving average and deviation.

The analysis of these series showed that the relative frequency distributions of overshooting time of all seven stations are close to normal (Gaussian) distribution. A distribution of one such station is presented in Fig. 5 (the width of columns in the histogram is constant and equal to 0.4s).

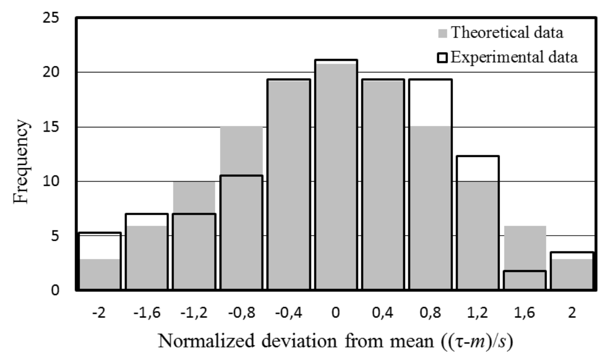

Fig. 5. The frequency distribution of the overshooting time.

III. Statistical Evaluation of Measurement Results

The aim of measuring the deviation overshooting time of FM stations with the aid of radio monitoring stations is to determine whether it exceeds the maximum permissible value τ_{max}. As overshooting time τ is a random variable, measurement results only allow for establishing that time τ exceeds the value τ_{max} with a certain probability.

A. The probability that overshooting time exceeds the limit (when the mean is known)

As presented above, time τ is a random variable with a distribution close to normal and its standard deviation s is directly related to the sample mean m:

$$s = k \cdot m . \quad (1)$$

In order to apply a mathematical apparatus, we shall consider that distribution of time τ is normal (Gaussian) with the expected value (mean) μ and the standard deviation σ, and there is a direct relation between these parameters:

$$\sigma = k \cdot \mu . \quad (2)$$

So we have the probability density function:

$$p(\tau) = \frac{1}{k \cdot \mu\sqrt{2\pi}} \exp\left[-\frac{1}{2} \cdot \left(\frac{\tau - \mu}{k \cdot \mu}\right)^2\right]. \quad (3)$$

By standardizing, we get a random variable z:

$$z = \frac{\tau - \mu}{k \cdot \mu} \quad (4)$$

with the standard normal distribution (independent of the parameters μ and σ) and cumulative distribution function $P(z)$:

$$P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{\xi^2}{2}\right) d\xi \ . \quad (5)$$

The variable z which corresponds to the given probability $P(z) = 1\text{-}\alpha$ shall be marked with the index z_α:

$$P(z_\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_\alpha} p(z)dz = Prob[z \leq z_\alpha] = 1 - \alpha \ . \quad (6)$$

It follows from equation (6) that the probability of a random variable z to exceed a particular level z_α is:

$$Prob[z > z_\alpha] = \frac{1}{\sqrt{2\pi}} \int_{z_\alpha}^{\infty} p(z)dz = 1 - P(z_\alpha) = \alpha \ . \quad (7)$$

Integral (7) cannot be evaluated analytically. Note that there are many ways (a table of the standard normal distribution, usually called as Z table; normal calculator, etc.) for the given probability $P(z_\alpha)$ to determine the corresponding value z_α or the opposite: for the given z_α to determine the corresponding probability $P(z_\alpha)$.

It is now possible to calculate the probability that the time τ will exceed the threshold τ_{max} and how this probability depends on the known mean μ. The expression for a normalized mean follows from equation (4):

$$\frac{\mu}{\tau_{max}} = \frac{1}{1 + k \cdot z_\alpha} \ , \quad (8)$$

where numbers z_α are determined by the corresponding values of α.

Calculated this way, the probability [*Prob* ($\tau > \tau_{max}$)] dependence on the normalized mean is shown in Fig. 6, where the value k=0.4 has been used.

B. *Statistical evaluation of measurement results*

In order to determine if time τ exceeds τ_{max} with a certain probability, we should know the "true" mean μ. In general it is not known, but if N measurement results of time τ were collected, then we would be able to determine the sample mean m. If the time τ distribution is normal, then the sample mean distribution is also normal. Therefore, the standard deviation of the sampling distribution of the sample mean is [2]:

$$s_m = \frac{\sigma}{\sqrt{N}} \ . \quad (9)$$

As before, by standardizing, we get a random variable z:

$$z = \frac{m - \mu}{\sigma\sqrt{N}} = \frac{(m - \mu)\sqrt{N}}{k \cdot m}. \quad (10)$$

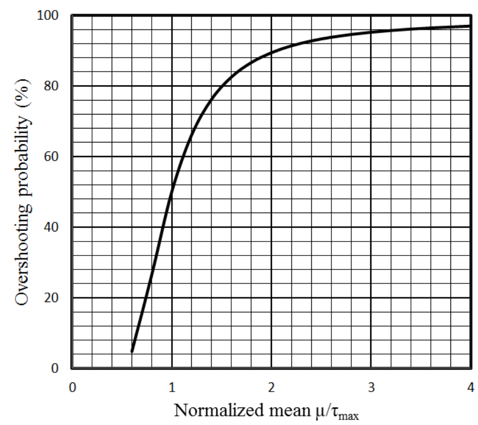

Fig. 6. Overshooting probability as function of overshooting time normalized mean.

It is now possible to calculate the lower endpoint μ_β, which is exceeded by mean μ with a confidence level β [2]. As mentioned above, the confidence level β corresponds to the number z_β. Therefore, it follows from equation (10) that:

$$z_\beta = \frac{(m - \mu_\beta)\sqrt{N}}{k \cdot m} \ . \quad (11)$$

From the equation (11) we obtain the relationship between the normalized parameter μ_β/τ_{max} and value m/τ_{max}:

$$\frac{m}{\tau_m} = \frac{\mu_\beta}{\tau_{max}} \cdot \frac{\sqrt{N}}{\sqrt{N} - z_\beta \cdot k} \ . \quad (12)$$

From combination of formulas (8) and (12), we obtain:

$$\frac{m}{\tau_m} = \frac{1}{1 + k \cdot z_\alpha} \cdot \frac{\sqrt{N}}{\sqrt{N} - z_\beta \cdot k} \ . \quad (13)$$

This equation allows to calculate the probability [*Prob* ($\tau > \tau_{max}$)] dependence on the normalized sample mean to a few fixed confidence levels. The calculation results when a sample mean is determined from one measurement of the time τ (therefore m=τ) are shown in Fig. 7, where the value k=0.4 has been used.

The curves presented in Fig. 7 allow evaluating the results of the deviation measurements. For example, let us assume that we have done a single measurement of time τ (therefore m=τ) and the measurement result is τ/τ_{max}=3. In this case, we could say that with a 90% confidence level the probability of time τ to overshoot the value τ_{max} would be equal to 79%. At an 80% confidence level this probability would be higher and equal to 95%.

If the measured value τ exceeds the $3\tau_{max}$, with a high confidence level the probability of time τ to overshoot the value τ_{max} is close to one. If the measured time τ value lies between τ_{max} and $3\tau_{max}$, it is suggested to increase the number of measurements N to three. The calculation results, when a sample mean is determined by measuring the value of time τ for three times (sample size N = 3), are shown in Fig. 8, where the value k=0.4 has been used.

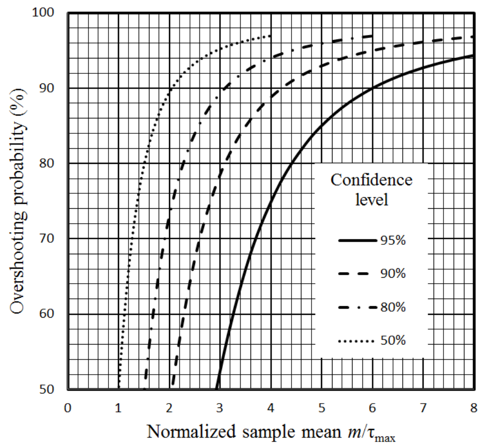

Fig. 7. Overshooting probability as a function of overshooting time normalized sample mean (sample size N=1).

In case sample size equals to 3, the conclusion that time τ overshoots the value τ_{max} can be done by significantly smaller normalized sample means.

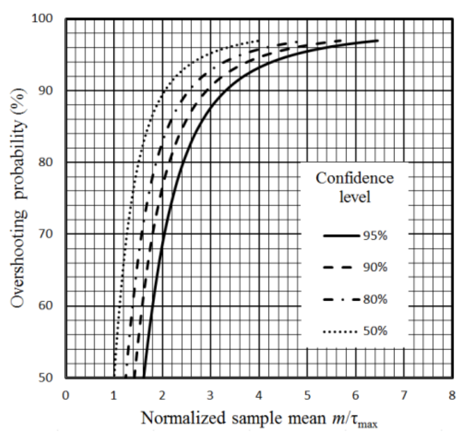

Fig. 8. Overshooting probability as a function of overshooting time normalized mean (sample size N=3).

It is clear that by increasing the sample size N, the sample mean m converges to the "true" mean μ. Therefore, all the curves that correspond to N = 10 practically merge with the curve corresponding to a 50% confidence level. Note that the curve corresponding to a 50% confidence level is nothing else but the probability for a random variable τ to overshoot limit value τ_{max}, as shown in Fig. 6.

C. *Enforcement threshold for overshooting time*

As presented above, if one measured value τ exceeds the $3\tau_{max}$, the probability of time τ to overshoot the value τ_{max} is close to one. This allows us to conclude that the value $3\tau_{max}$ can be considered an enforcement threshold.

In order to test the enforcement threshold efficiency, the measurements of one FM station were made within several days. Fig. 9 presents the measurement results of time τ (in total 97 sample values). The sample mean m of this series has been calculated. As the series length is quite long, it can be argued that it practically coincides with the true mean μ. The situation was modelled where the true mean μ coincides with the enforcement threshold $3\tau_{max}$. The enforcement threshold and overshooting time limit τ_{max} are presented in red lines in Fig. 9. It was mentioned above (Fig. 6) that in such situations 5% of measurement values shall be less than τ_{max}, and the rest of 95% of values – higher than τ_{max}. 5% of 97 values of time τ are equal to 4.85. As seen in the Fig. 9, 4 values are below τ_{max} level. Such a favourable coincidence confirms once more that the distribution of time τ is close to normal.

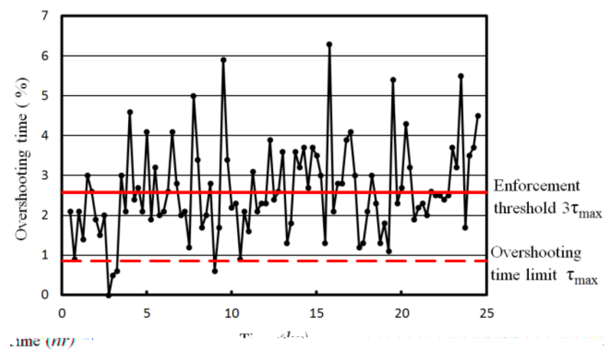

Fig. 9. Enforcement threshold of the test result.

It is clear that the enforcement threshold $3\tau_{max}$ is useful for the Regulator. When the Regulator produces a notification of violation to an Operator, it must be ensured that the measured time τ **exceeds** the specified value.

If a broadcast operator tests deviation compliance with the set conditions, it must be sure that the measured time τ does **not exceed** the indicated value. In this case, it would be logical to introduce **a guard threshold**. The calculation procedure could be analogous to the enforcement threshold calculation procedure.

IV. Conclusion

This article proposes a method that allows a more objective assessment of frequency deviation of the FM stations. It also gives the possibility to calculate the enforcement threshold for overshooting time value which is much more convenient and likely to be more accurate than its empirical determination.

Communications Regulatory Authority has been using the above mentioned method for three years already. During this period many notifications of violation were produced to operators; several operators have been fined, but none of the decisions on overshooting the deviation was denied.

References

[1] Recommendation ITU-R SM.1268-3 "Method of measuring the maximum frequency deviation", Radiocommunication Sector of ITU, August 2014.

[2] J.S. Bendat and A.G. Piersol, Random Date. Analysis and Measurement Procedures, AWiley-Interscience Publication, 1986.